1 (a) Write $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$7 + 12x - 3x^2$

$= -3(x^2 - 4x) + 7$ (1)

$= -3[(x-2)^2 - 4] + 7$ (1)

$= -3(x-2)^2 + 12 + 7$ (1)

$= -3(x-2)^2 + 19$

Arrange in the form of $a + b(x+c)^2$

$= 19 - 3(x-2)^2$ (1) where $a = 19$

$b = -3$

$c = -2$

$19 - 3(x-2)^2$

(4)

The curve **C** has equation $y = 7 + 12x - 3x^2$
The point A is the turning point on **C**.

(b) Using your answer to part (a), write down the coordinates of A.

$y = 19 - 3(x-2)^2$

y-coordinate (when $x = 2$, $y = 19$)

x-coordinate ($x - 2 = 0 \rightarrow x = 2$)

(2 , 19) (1)

(1)

(Total for Question 1 is 5 marks)

2 The function f is such that $f(x) = 5 + 6x - x^2$ for $x \leqslant 3$

(a) Express $5 + 6x - x^2$ in the form $p - (x - q)^2$ where p and q are constants.

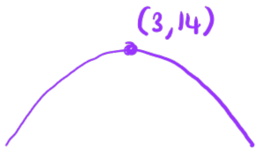
$-x^2 + 6x + 5$

$-(x^2 - 6x - 5)$

$-\left[(x-3)^2 - 9 - 5\right]$ 1

$-(x-3)^2 + 14$

$\therefore\ 14 - (x-3)^2$ 1 where $p = 14$, $q = 3$



$14 - (x-3)^2$

(2)

(b) Using your answer to part (a), find the range of values of x for which $f^{-1}(x)$ is positive.

$f(x) = 14 - (x-3)^2$

Range of $f^{-1}(x)$

$y \leq 3$

Let $f(x) = y$: $y = 14 - (x-3)^2$ 1

Find x in terms of y

$y = 14 - (x-3)^2$

$y - 14 = -(x-3)^2$

$(x-3)^2 = 14 - y$

$x - 3 = \pm\sqrt{14-y}$

$x = 3 \pm \sqrt{14-y}$ 1

$f^{-1}(x) = 3 - \sqrt{14-x}$ — since y should be ≤ 3

1

If $f^{-1}(x) > 0$

$3 - \sqrt{14-x} > 0$ 1

$3 > \sqrt{14-x}$

$9 > 14 - x$

$x > 5$

or

$3 - \sqrt{14-x} \leq 3$

$0 \leqslant \sqrt{14-x}$

$x \leqslant 14$

$\therefore$ Hence, $5 < x \leq 14$ 1

$5 < x \leq 14$

(5)

(Total for Question 2 is 7 marks)

3 (b) Express $x^2 - 10x + 40$ in the form $(x + a)^2 + b$, where a and b are integers.

By using completing the square method :

$(x-5)^2 - 25 + 40$ 1

$= (x-5)^2 + 15$ 1

where $a = -5$

$b = 15$

$(x-5)^2 + 15$

(2)

(Total for Question 3 is 2 marks)

4 A particle P is moving along a straight line.
The fixed point O lies on the line.

At time t seconds ($t \geqslant 0$), the displacement of P from O is s metres where

$$s = t^3 - 9t^2 + 33t - 6$$

Find the minimum speed of P.

speed, $v = \frac{ds}{dt} = 3t^2 - 18t + 33$ (1)

$v = 3(t^2 - 6t + 11)$

By completing the square:

$v = 3[(t-3)^2 - 9 + 11]$ (1)

$= 3[(t-3)^2 + 2]$ (1)

$v = 3(t-3)^2 + 6$ (1)

v is at minimum when first term $= 0$ (cannot be negative because of square)

when $t = 3$, $v = 3(3-3)^2 + 6$

$= 0 + 6$

$= 6$ (1)

$\therefore$ minimum speed of P is $6\ ms^{-1}$.

6 m/s

(Total for Question 4 is 5 marks)

5 (a) Express $2x^2 - 12x + 3$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$2(x^2 - 6x) + 3$ (1)

$2[(x-3)^2 - 9] + 3$ (1)

$2(x-3)^2 - 18 + 3$

$2(x-3)^2 - 15$ (1)

where $a = 2$, $b = -3$ and $c = -15$

$2(x-3)^2 - 15$
....
(3)

(Total for Question 5 is 3 marks)

6 Express each of a, b and c in terms of q so that

$$q + 12x - qx^2$$

can be written as $a - b(x - c)^2$

$$-q\left(x^2 - \frac{12}{q}x\right) + q \quad (1)$$

$$-q\left[\left(x - \frac{12}{2q}\right)^2 \ldots\right] + q \quad (1)$$

$$-q\left(x - \frac{6}{q}\right)^2 + \frac{36}{q} + q \quad (1)$$

$$a = \frac{36}{q} + q$$

$$b = q \quad (1)$$

$$c = \frac{6}{q}$$

$a =$ $\frac{36}{q} + q$

$b =$ q

$c =$ $\frac{6}{q}$

(Total for Question 6 is 4 marks)

Given that a, b and c are integers,

7 (b) express $3x^2 + 12x + 19$ in the form $a(x + b)^2 + c$

$3(x^2+4x)+19$ (1)

$3[(x+2)^2-4]+19$

$3(x+2)^2-12+19$

$3(x+2)^2+7$ (1)

$a=3$, $b=2$, $c=7$

$3(x+2)^2+7$

....

(2)

(Total for Question 7 is 2 marks)

8 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$7 - 3(x^2 - 4x)$ (1)

$7 - 3[(x-2)^2 - 4]$ (1)

$7 - 3(x-2)^2 + 12$

$19 - 3(x-2)^2$ (1)

$19 - 3(x-2)^2$

(3)

C is the curve with equation $y = 7 + 12x - 3x^2$
The point A is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of A

(1)

(2 , 19)

(1)

(Total for Question 8 is 4 marks)

The function g is such that

$$g(x) = 5x^2 - 20x + 23$$

9 (c) Express g(x) in the form $a(x - b)^2 + c$

$g(x) = 5(x^2 - 4x) + 23$ 1

$= 5[(x-2)^2 - 4] + 23$ 1

$= 5(x-2)^2 - 20 + 23$

$= 5(x-2)^2 + 3$ 1

$5(x-2)^2 + 3$

(3)

(Total for Question 9 is 3 marks)

10 Express $3x^2 - 6x + 5$ in the form $a(x - b)^2 + c$

$3(x^2 - 2x) + 5$ (1)

$= 3[(x-1)^2 - 1] + 5$

$= 3(x-1)^2 - 3 + 5$ (1)

$= 3(x-1)^2 + 2$ (1)

$3(x-1)^2 + 2$

(Total for Question 10 is 3 marks)